Answers to Coursebook questions – Chapter E5

1 A black hole would have 2 immediate signatures.
Mass falling into the black hole would accelerate, heat up and radiate, also in the X-ray region. The black hole would also strongly influence the orbits of any nearby stars.

2 The total energy radiated would be $3.9 \times 10^{26} \times 10^{10} \times 365 \times 24 \times 3600 = 1.2 \times 10^{44}$ J.

The corresponding mass is $m = \frac{E}{c^2} = \frac{1.2 \times 10^{44}}{9.0 \times 10^{16}} = 1.4 \times 10^{27}$ kg.

This is a small fraction $\frac{1.4 \times 10^{27}}{2.0 \times 10^{30}} \approx 10^{-3}$ of the present mass of the sun.

3 **a** The speed is given by $v = \frac{2\pi R}{T} = \frac{2\pi \times 10 \times 10^3}{10^{-3}} = 6.3 \times 10^7$ m s^{-1}.

b $\frac{6.3 \times 10^7}{3.0 \times 10^8} = 21\%$.

4 The star with one solar mass consumes energy slower **relative to its mass** and so will last longer.

5 A planetary nebula refers to the explosion of a red giant star that ejects most of the mass of the star into space. Not all planetary nebulas (of the 3000 or so that are known to exist) appear as rings the way the famous helix and ring nebulas appear. Even the ring-like ones appear ring-like because the gas surrounding the centre is very thin. A line of sight through the outer edges of the nebula goes through much more gas than a line of sight through the centre. Hence the centre looks transparent, while the edges do not.

6 With disbelief, since gold is heavier than iron and so cannot be formed by nuclear fusion in the core of a star.

7 In rough terms the energy available for fusion is the entire mass M of the star.
The energy that theoretically can be released is then $E = Mc^2$.

Since $L = \frac{E}{T}$, it follows that $kM^4 = \frac{Mc^2}{T}$, i.e. $T \propto \frac{1}{M^3}$.

So for a star of mass double that of the sun, the lifetime T is less than that of the sun by a factor of $2^3 = 8$.

8 It would be hard to believe, as the stars being O stars would have enormous luminosity and so would very quickly leave the main sequence since they consume energy too fast. The age of 100 million years is too long for these stars.

9 No, because all the elements that are necessary for life were made either in nuclear fusion processes in the cores of very heavy stars or during the supernova stage, when nuclei were irradiated with neutrons (to make the elements heavier than iron).

10 **a** A one solar mass star would evolve to become a red giant. As the star expands in size into the red giant stage, nuclear reactions inside the core of the star are able to produce heavier elements than helium because the temperature of the core is sufficiently high. The red giant star will explode as a planetary nebula ejecting most of the mass of the star into space and leaving behind a dense core. The core is no longer capable of nuclear reactions and the star continues to cool down. The core is stable under further collapse because of electron degeneracy pressure.

b The core has a mass that is definitely less than the Chandrasekhar limit and so ends up as a white dwarf.

11 **a** The mass difference is $4.00260 + 8.0053 - 12.0000 = 7.9\times10^{-3}\ \text{u}$, and so the energy released is $7.9\times10^{-3}\times931.5 = 7.36\ \text{MeV}$.

b The mass difference is $4.00260 + 12.0000 - 15.9941 = 8.5\times10^{-3}\ \text{u}$, and so the energy released is $8.5\times10^{-3}\times931.5 = 7.92\ \text{MeV}$.

12 Because the electric charge of helium nuclei is greater than that of hydrogen nuclei and so more energy is required in order to overcome the electrostatic potential energy between the nuclei. This requires a greater temperature.

13 **a** A planetary nebula is caused by the explosion of a red giant star.

b A supernova is caused by the explosion of a super red giant star.

14 The Chandrasekhar limit is a mass of about 1.4 solar masses. It represents the largest mass a white dwarf star can have. If the mass that is left behind after a planetary nebula or supernova is less than this limit, the star will become a stable white dwarf star.

15 A red giant forms out of a main sequence star when a certain percentage of the hydrogen of the star is used up in nuclear fusion reactions. The core of the star collapses and this releases gravitational potential energy that warms the core to sufficiently high temperatures for fusion of helium in the core to begin. The suddenly released energy forces the outer layers of the star to expand rapidly and to cool down. The star thus becomes a bigger but cooler star – a red giant.

16 See **Figure E5.9** (see page 529 in *Physics for the IB Diploma*), where the curve starts somewhat lower on the main sequence. A 5 solar mass main sequence star has a luminosity that is about $5^{3.5}\approx 300$ times the luminosity of the sun.

17 The total kinetic energy of the two helium nuclei is $2\times\frac{3}{2}kT$.

This equals the electric potential energy $\frac{1}{4\pi\varepsilon_0}\frac{(2\text{e})^2}{d}$. Hence $3kT = \frac{1}{4\pi\varepsilon_0}\frac{(2\text{e})^2}{d}$.

Numerically this is $T = 9\times10^9\,\frac{(2\times1.6\times10^{-19})^2}{3\times1.38\times10^{-23}\times10^{-14}}\approx 2\times10^9\ \text{K}$.

18 See **chapter 1.5 exercise 6** (see page 37 in *Physics for the IB Diploma*).

19 The blueshift implies that the surface of the star is moving towards us, i.e. the star is expanding. As the surface area of the star expands, the luminosity increases. From $T\rho^2 = \text{constant}$, and stars of the same mass, we have that $T = R^6 \times \text{constant}$. Since the larger the radius the larger is the luminosity, this shows that larger luminosity is obtained from the longer period stars.

20 The gas law states that $PV = nRT$. The number of moles is equal to $n = \frac{N}{N_A}$, where N_A is Avogadro's number. The Boltzmann constant k is defined by $k = \frac{R}{N_A}$ and so the gas law may be written as $PV = NkT$. Since $V \propto R^3$ and $N \propto M$ we have that $PR^3 \propto MT$. From dimensional analysis, equilibrium demands that $PA \approx \frac{GM^2}{R^2}$, where A is the area on which the pressure P acts. Since $A \propto R^2$ it follows that $P \propto \frac{M^2}{R^4}$ and combining these two equations we get $T \propto \frac{M}{R}$. This shows that as the star shrinks (the radius gets smaller) the temperature goes up. Now, the luminosity is given by $L = \sigma AT^4 \propto R^2\left(\frac{M}{R}\right)^4 = \frac{M^4}{R^2}$. And since $\rho \propto \frac{M}{R^3}$, i.e. $R \propto \left(\frac{M}{\rho}\right)^{1/3}$, it follows that $L \propto \frac{M^4}{M^{2/3}} = M^{3.3}$, the mass–luminosity relation!

21 The young star is on the main sequence and so mainly converts hydrogen to helium. A much older star of the same mass would have evolved (perhaps off the main sequence) and would now be producing much heavier elements than helium.

22 One of the proposed mechanisms involves black holes that heat up matter falling into them. The hot matter radiates a lot, giving the quasar its large luminosity.

23 They showed unusually large redshifts, indicating large recessional speeds at very large distances from us.

24 The radiation from the pulsar is emitted within a very thin cone whose axis is the direction of the magnetic field of the star. If this direction is different from the axis of rotation, then an observer will be getting radiation only when the axis of the cone is along the line of sight. This will only happen once during a revolution and so the radiation received will be pulsed.

25 Much larger.

26 **a** The time would be $\frac{2\times10^{11}}{10^3} = 2\times10^8$ y.

b At this rate quasars would consume a significant fraction of the mass of the universe. Since this is not the case, quasars cannot live for long.

